IMC Selection Test, May 2026
Organized by
Bhaskaracharya Pratishthana, Pune
DAY-1

Date: 09/05/2026 Max. Marks: 50
Time: 8.30 AM to 1.30 PM
N.B.: Each question carries 10 marks.

1. Let $f : \mathbb{R}^n \to \mathbb{R}$ be defined by
$$f(x_1, \ldots, x_n) = \frac{x_1^{a_1} x_2^{a_2} \cdots x_n^{a_n}}{x_1^{b_1} + \cdots + x_n^{b_n}} \text{ for } (x_1, \ldots, x_n) \neq (0, \ldots, 0) \text{ and } f(0, \ldots, 0) = 0,$$
where a_i's are positive integers and b_i's are even positive integers. Determine precisely all the a_i's and b_i's for which f is discontinuous at 0.

2. Let G be a group in which exactly two elements (different from the unit element) are commuting. Show that G is isomorphic to either $\mathbb{Z}_3$ or S_3.

3. Let $a_0, b_0 \in [0, 1], a_n = \displaystyle\int_0^1 \max\{x, b_{n-1}\}dx$ and $b_n = \displaystyle\int_0^1 \min\{x, a_{n-1}\}dx, n \geq 1$. Find the limit of the sequence $\{a_n + b_n\}$, if it exists.

4. Define a function $f : \mathbb{R} \mapsto \mathbb{R}$ as follows:
   (a) $f(p)$ = product of greatest and least roots of $q(x) = x^3 - 3x - p$, if $q(x)$ has three real roots.
   (b) $f(p)$ = the square of the real root if $q(x)$ has only one real root.

   Determine the minimum value of $f(p)$ as p varies over all real numbers.

5. What are all possible ways to fill in the blanks with single digits such that the following sentence is true?
"In this sentence, the number of occurrences of the digit 0 is ____, of the digit 1 is ____, of the digit 2 is ____, of the digit 3 is ____, of even numbers is ____, of odd numbers is ____, and of prime numbers is____."

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DAY-2

Date: 10/05/2026 Max. Marks: 50
Time: 8.30 AM to 1.30 PM
N.B.: Each question carries 10 marks.

1. Let $\{x_n\}$ be a sequence of real numbers defined by $x_1 = 1$ and
$$x_{n+1} = \frac{n^2}{x_n} + \frac{x_n}{n^2} + 2$$
for all $n \geq 1$. Prove that $\lim\limits_{n\to\infty}(x_n - n) = \frac{1}{2}$.

2. Anand is going to play two games of chess with an opponent whom he has never played against before. The opponent is equally likely to be a beginner, intermediate, or a master. Depending on which, Anand's chances of winning an individual game are $90\%, 50\%$ or 30%, respectively. Assume that, given the skill level of the opponent, the outcomes of the games are independent. Given that Anand wins the first game, what is the probability that he will also win the second game?

3. Let us write $\mathbb{Z}_{12} = \{0, 1, \ldots, 11\}$. A triad (a, b, c) is a three-tuple of distinct elements of $\{0, 1, \ldots, 11\}$. Two triads (a, b, c) and (a', b', c') are defined to be equivalent either if $(a, b, c) = (a', b', c')$ or if there is $k \in \mathbb{Z}$ such that $a' \equiv (a + k) \pmod{12}$, $b' \equiv (b + k) \pmod{12}$, $c' \equiv (c + k) \pmod{12}$. Find the number of equivalence classes of triads.

4. Let $f(x)$ be a monic polynomial of degree n with distinct real zeros, say $a_1 < a_2 < \cdots < a_n$. Let $\delta(f) = \min(a_i - a_{i-1})$. Show that $\delta(f') > \delta(f)$.

5. (a) Let A, B be 2×2 matrices with entries complex numbers. If $A^2 + B^2 = 2AB$, prove that $AB = BA$ and Trace (A) =Trace (B).
   (b) If the entries of A and B are from $\mathbb{Z}_2$, and if $A^2 + B^2 = 2AB$, is it true that $AB = BA$? Justify.
   (c) If A, B are 3×3 matrices with entries complex numbers satisfying $A^3 - B^3 = 3(A^2B - AB^2)$, is it true that $AB = BA$? Justify.
